LIFE IS NOT A FRACTAL IN HILBERT'S SPACE

LA VIE EST PAS UNE FRACTALE DANS L'ESPACE DE HILBERT

VIAŢA NU ESTE UN FRACTAL IN SPAŢIUL HILBERT

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Abstract

This paper focuses at one of Ioan Petru Culianu main idea: Religion is an ideal object, which means it is a fractalic system. To decipher what exactly this assertion of Culianu means, this study will track it back toward the sources of Culianu, briefly examining the concept of 'fractal' in Benoit Mandelbrot's theory and its processing in Rudy Rucker's view. After presenting the point of view of Culianu, this article will end with an analysis on the selected topic.

Résumé

Ce document met l'accent sur une idée principale qui appartient à Ioan Petru Culianu: la religion est un objet idéal, ce qui signifie qu'elle est un système fractal. Pour déchiffrer exactement ce que cette affirmation de Culianu signifie, cette étude permettra de le retourner vers les sources de Culianu, en examinant brièvement le concept de « fractal » dans la théorie de Benoit Mandelbrot et de son traitement dans la vision de Rudy Rucker. Après avoir présenté le point de vue de Culianu, cet article se termine par une analyse sur le sujet sélectionné.

Rezumat

Această lucrare se concentrează pe una dintre ideile principale ale Ioan Petru Culianu: Religia este un obiect ideal, ceea ce înseamnă că este un sistem fractalic. Pentru a descifra ce reprezintă exact această afirmaţie a lui Culianu, studiul de faţă va urmări sursele lui Culianu, examinând pe scurt conceptul de „fractal" în teoria lui Benoit Mandelbrot şi prelucrarea acestuia în viziunea lui Rudy Rucker. După prezentarea punctului de vedere propriu lui Culianu, acest articol se va încheia cu o analiză pe tema selectată.

Keywords: *Ioan Petru Culianu; Rudy Rucker; Benoit Mandelbrot; fractal; ideal object*

Mots-clés: *Ioan Petru Culianu; Rudy Rucker; Benoit Mandelbrot; fractale; objet idéal*

Cuvinte-cheie: *Ioan Petru Culianu; Rudy Rucker; Benoit Mandelbrot; fractal; obiect ideal*

1. Introduction

The most important characteristic of human mind is probably the imagination. There is virtually nothing that the mind cannot imagine. Let's think of one of the oddest 'thing' we could ever imagine: the infinity. If I take a stick, I can cut it in two halves. One of them, I can cut in half again. Then, again; and again. And so on until the piece is so small than I can hardly see it, not cut it. I can only give up. But, in my mind, the situation is completely different. If I have a segment of a line

AB, I can 'cut' it in two: AA_1 and A_1B. AA_1 can be split in two. The same operation can go forever. Or, using numbers: mathematically I can divide the number 1 in two: 1/2, then in half = 1/4; and in half = 1/8; and so on: I can continue this forever. In other words, I can do it infinitely. The beauty of mathematics is that it benefits from the imagination at maximum. The infinity is probably one of the most beautiful discoveries of math or philosophy (i.e. of our culture), in other words, of human mind. Meditating on 'infinity' Mandelbrot came with a mathematical theory of beautiful things named 'fractals'. Rucker meditated on fractals and came with the idea of 'life as a fractal'. Culianu read Rucker's books and hypothesized that mind's "products", as Philosophy, Literature, Science, and Religion, are 'fractalic ideal objects'.

Willing to find out what "ideal objects" are, the researcher had to start in order, with the first one, and followed the "fractal" tracks.

2. Benoit Mandelbrot (1924-2010)

Few years before coming with the term of "fractal", Mandelbrot wrote an article about the coastline of England, on the theme known as "coastline paradox" (MANDELBROT, 1967, 636-638). The main idea of this is: the length of the coastline depends on the scale of measurement. That is, if one measures the coastline using a unit of 100 km, the Great Britain's coastline is approx. 3200 km long; using a unit of 10 km, it is longer: about 3400 km. If we consider all the irregularities, ridges and shapes of the coastline, what result will obtain one who measures it with a 10 cm ruler? To the limit, it have been affirmed that the measured length increases without limit as the measurement scale decreases towards zero.

In the following years after publishing the article, Mandelbrot developed his theory of 'fractals', a coastline paradox solver. Briefly, a "fractal" is a geometrical figure that divided in parts displays a recurring pattern (called self-similar) at every scale, no matter how small. The term is taken from the Latin *fractus*, meaning *fragmenting*. So a fractal is a pattern that repeats itself, perpetually. Self-similarity can be for example: *exact* self-similarity, *quasi* self-similarity, or *statistical* self-similarity, which means repeating a pattern stochastically (or randomly, in probability theory) thus statistical measures are maintained throughout scales.

Mandelbrot declares in his annotated edition of his book (MANDELBROT, 1983, 1): "I conceived and developed a new geometry of nature and implemented its use in a number of diverse fields. It describes many of the irregular and fragmented patterns around us, and leads to full-fledged theories, by identifying a family of shapes I call fractals. The most useful fractals involve chance and both their regularities and their irregularities are statistical." On many websites anyone can see now fractals; many of them are more than interesting: they are beautiful.

The above ideas are integrated even in British Cartographic Society's website. When one asks: "How long is the UK coastline?" the answer is: "It depends on the scale at which you measure it". Even the notion of "fractal" is used now: "This is an example of fractal behaviour, and the result is that the only truly 'accurate' measure of the length of the coastline — any coastline — shows that it is almost infinitely long."[1]

Of course, in practice we have to approximate the length of the coastline (or the borders, and so on). No one will ever measure the length of a coastline with a microscope. In mathematics there is no conflict in considering the coastline as infinite. But is it really infinite (or, "almost infinitely long") in reality? I think this is a philosophical problem, which takes as back to Zeno's paradox, which says that I cannot make a step, because first I have to make half of it, and half of the half, and so on infinitely. Mathematically, he is right. But in fact I can walk all right. Similarly, if the coastline were infinitely in reality, no person would be able to go around it with a boat, for example, which was not the case.

Are all fractals infinite? As far as I understood, mathematically the pattern's fragmentation goes forever, so fundamentally all fractals are infinite. In one of the clearest books on fractals,

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[1] See http://www.cartography.org.uk/default.asp?contentID=749

Helmberg answers the question "Are there also fractal curves of finite length?" in this way: "If we want to keep the length of A(k) bounded, our only chance is to cut the unit interval into finitely many pieces and to place them disjointly from each other. This will destroy continuity in the resulting fractal, but we have already made peace with this fact when dealing with the CANTOR set." (HELMBERG, 2007, 35). Thus, only if we "intervene" and stop fractal "development" we can talk about a "finite fractal" by this understanding a segment (a piece) of the (infinite) fractal.

3. Rudy Rucker (b. 1946)
Rucker defines fractals as "infinitely detailed, perfectly self-similar shapes"; mathematical shapes, of course. He identified this issue, too, so he affirms without equivoque:

> A difference between a Koch curve and an actual coastline is that the Koch curve is an idealized mathematical form with structure at *infinitely* many levels. Just as we do not think of a real tree as having infinitely many forks, we do not think of a physical coastline as really being an infinitely complex line, but for many purposes, a fractal is a better model than is a finitely complex smooth curve. (RUCKER, 1987, 174-175)

Indeed, using fractals makes a lot of things easier for specialists in many domains, including the quoted Cartography. So, in terms of usefulness, is better to use fractals. But is it also good to extrapolate it in other domain, like humanities? For example, in Philosophy, or in Culianu's cases, in Religion? Or, even more, in Rucker's assertion, to life itself?

Apart from the metaphorical value, one could ask when reading the next affirmation of Rucker: "life is a fractal in Hilbert space" if it has any meaning. Different from quantum mechanics, for Rucker 'Hilbert space' means a space with endless dimensions. Hilbert space is an abstract concept that goes beyond the two-dimensional plane and the three-dimensional space, to a space with dimensions of infinite number. This is a mathematical space. I do not see any argument for considering it real.

Think now of all the humans living nowadays, and have lived since the beginning of Earth; there is an estimation of Population Reference Bureau that the number is around 108 billion[2]. Assume an average of numbers of years lived of 70 years. This is a generous one, knowing that since the 19th century life expectancy was around 30-40. In seconds, this is about $3.2x10^5$, suppose that one has 10 thoughts per second. Current estimation is somewhere between 1 and 2, but let's assume those are only conscious thoughts, and the rest are subconscious. My life is what I am thinking, and doing accordingly. In total is maximum $3.2x10^6$. All the possible thoughts, of all people that ever lived would be maximum $3.2x10^6x108x10^9$, so it is approximate $3x10^{17}$. In Rucker's "life box" there are included all possible connections (RUCKER, 1987, 180). Connecting everything with everything (this is the maximum of all possible connection) the result will be about $(3x10^{17})^{17}$. Approximating, it is equal to $1.3x10^{298}$. This number is quite big. Suppose humans will live another 100 billion year from now (or 100 hundred billion, or a 100 billion of billion, it makes no difference). My intention is not to calculate in an exact manner like mathematicians, but to show that the result is *a lot*, indeed, but still far, far away from infinity.

After reading all the magnificent math presentations that prove the world is not of endless complexity; and the confession of the author: "I can't prove that the universe is more complex than about three billion bits. This isn't what I expected, but it's the truth" (RUCKER, 1987, 290), how can one agree with the affirmation that life is a fractal in Hilbert space? This means: infinitely self-similar repetition of the same pattern in an endless space. Firstly, life is not infinite. Secondly, life is not self-similar. Thirdly, life is real (at least in our view, not in people's who think it is a sort of illusion, e.g. *Maya*), and Hilbert space is a mathematical concept.

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[2] See http://www.prb.org/Publications/Articles/2002/HowManyPeopleHaveEverLivedonEarth.aspx

4. Ioan Petru Culianu (1950-1991)

Culianu was in search for a scientific theory to sustain his cognitive hypothesis on Religion as an ideal object. Rucker's and Mandelbrot's theories looked very much appropriated so he used them.

> Ideal objects are systems operating in a logical dimension and cannot go beyond their (generally quite simple) premises. Systems are fractalic in nature, that is, they tend to produce solutions ad infinitum according to (simple) production rules. And they interact with each other in quite strange ways, forming other systems whose general pattern of uncanny complexity may be called history. (CULIANU, 1992, 21)

The principle of generating and the "runtime" from these theories sustain Culianu's model[3]. But as we saw, sometimes mathematical theories do not fit in "life", i.e. in what human have thought and have done. One part of this is Culianu's main domain of expertise, Religion.

Religion is not a fractal because it lacks the self-similarity principle; and it could not be infinite, either. Religion could be instead seen as a finite system, running in human minds similar to a computer program: started from simple premises it produces (predictable) 'solutions' in time; some remained virtual, other manifested in history under different names. Culianu was right about the system, but he was wrong about the fractal. This does not make his model less appealing. It just needs some adjustments, and probably a new scientific theory as foundation.

5. Conclusions

Let's start the final discussion with another example, less abstract. I can easily imagine that I am writing something; but I want to do it forever: to keep it simpler, let's choose the letter 'a'. It does not have to have any sense; but nothing can stop us to think of this possibility. Because my mind has the power of thinking infinitely the infinity, in my imagination I can do that. But if I want to prove that I can do the same in real life, in my materialistic and practical life, the things are changing. Suppose I start doing this job (and get paid for it by a meaningless institution). I write the letter 'a', let's say one per second. In one hour I have already written 3600 'a', in ten hours of hard work I have done 36000. I'm a hard worker so I don't stop yet. I will continue this for eight more hours; maybe I will get double pay for extra hours. But after 18 hours I must stop, because I have to go to sleep. The next day I will wake up with new forces and I will start doing my job with joy. For my goal is the infinity. Could this work be done infinitely? Unfortunately I cannot do it, since at some point, I hope very far from now, my life will end. But there is hope: my children, and their children and the children of the children of the children, and so on, can continue my work. Maybe we all together can achieve the infinity? Well, I am afraid that will not be possible regardless all my followers will work on that. At some point the Earth will cease to exist. And even if humans will move to another planet, at some point this will end, too. Nobody knows for sure, but very consistent scientific theories, to which I subscribe, claim that the Universe itself will not exist forever. I am sorry to repeat it, and to contradict again my reputable companions in the unknown of the mind, Rucker and Culianu, but life is not a fractal in Hilbert space. Only my imaginary life is. The imagination itself is (or could be considered for the sake of the metaphor) a fractal in Hilbert space. Likewise, as Rucker himself agrees, a tree is not a fractal: only a mathematical tree is. The Tree of Gnosis is not an infinite system; it is a finite one.

So one can think of something infinite, but cannot transpose it into life. This must have been so frustrating for our ancestors, who didn't have computer games to entertain their minds. So what happened when they were thinking of the Infinity itself? They imagined a transmutation of the infinity into the finiteness: this is what we had named long time ago, and is still known today as Religion. Apparently God was the only answer in finding a solution to the Infinity problem. Since our lives are limited, he was the only one who could apply for this job position. What happen in the meantime is known as History of Religion. Nowadays, this is a pain in the mind for scholars. Many renounced at this name; moreover, some abandoned the History itself. They approach Religion from

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[3] For more details, on Culianu's model, see DAVID, 2014; for its application in Philosophy, see DAVID, 2015.

many other domains: Economy, Sociology, Psychology, of course Philosophy, and so on, when there is only one proper approach: the cognitive one. How to analyze the imagination, the mind and the ideas? Luckily, humans love to leave traces of their thoughts, so there are plenty of images, sculptures, constructions and texts to research. So we don't have to worry: we still have the object of our work, and we will not end up unemployed any soon.

Certainly, this is taking us back to History. It is indispensable, in order to discover the 'documents' (i.e. all the tracks we left behind from our imagination in a form or another) and to put them in the right place they belong. Back to Hermeneutics, because we have to meditate on, analyze, interpret and understand those documents. And also back to all the other sciences, already mentioned or unnamed here, like neuro-sciences (we'll include here Neuro-philosophy, too): they are all of huge help. The Study of Religion cannot afford to refuse any support the other sciences offer. After all, as Culianu said, they all talk about the same thing; they have all the same source, the human mind. They are all maps of our mind. Applying Culianu's model in religion, philosophy, science and literature, we probably will properly unveil many of the links between our 'creations' (either of humanities, sciences, or arts) and reconsider our place on Earth, the relations between all of us, and our religions, philosophies or beliefs.

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